

[Problem solution by applying MacLaurin's Series]

MacLaurin's Series is given by

$$\bullet f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \dots \text{to } \infty.$$

Q. (1) Expand $\cos x$ by MacLaurin's theorem.

Solution:— Here, let $f(x) = \cos x$

Differentiating w.r.t x , successively, we get

$$f'(x) = -\sin x, f''(x) = -\cos x, f'''(x) = \sin x$$

and so on.

on putting $x=0$, we have

$$f(0) = 1, f'(0) = 0, f''(0) = -1, f'''(0) = 0, \dots$$

Now, by MacLaurin's theorem, we, have

$$f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \dots$$

$$\therefore \cos x = 1 + x \cdot 0 + \frac{x^2}{2!} \cdot (-1) + \frac{x^3}{3!} \cdot 0 + \dots$$

$$\text{i.e. } \cos x = 1 - \frac{x^2}{2!} + \frac{x^4}{4!} - \frac{x^6}{6!} + \dots \text{ to } \infty.$$

Q. (2) — Expand $\log(1+e^x)$ in ascending powers of x containing x^4 .

Solution:— Here, $f(x) = \log(1+e^x)$

$$\text{Then, } f(0) = \log(1+e^0) = \log(1+1) = \log 2$$

[\because the value of e lies between 2 and 3]

$$f'(x) = \frac{1}{1+e^x} \cdot e^x \Rightarrow f'(0) = \frac{1}{2}$$

$$f''(x) = \frac{(1+e^x)e^x - e^x(0+e^x)}{(1+e^x)^2} = \frac{e^x + e^{2x} - e^{2x}}{(1+e^x)^2}$$

$$\Rightarrow f''(x) = \frac{e^x}{(1+e^x)^2}$$

$$\therefore f''(0) = \frac{1}{4}$$

$$f'''(x) = \frac{(1+e^x)^2 e^x - e^x(2(1+e^x) \cdot (0+e^x))}{(1+e^x)^4}$$

$$= \frac{e^x(1+e^x)[e^x + e^{2x} - 1 + e^2 - 2e^x]}{(1+e^x)^4}$$

$$= \frac{e^x(1-e^{2x})}{(1+e^x)^4} = \frac{e^x(1-e^x)}{(1+e^x)^3}$$

$$\therefore f'''(0) = \frac{1(1-1)}{(1+1)^3} = \frac{0}{2^3} = 0$$

proceeding in similar manner, we can find

$$f^{iv}(0) = -\frac{1}{8} \text{ and so on.}$$

Thus, we have, $f(x) = \log(1+e^x)$, $f(0) = \log 2$

$$f'(0) = \frac{1}{2}, f''(0) = \frac{1}{4}, f'''(0) = 0, f^{iv}(0) = -\frac{1}{8}$$

and so on.

Hence, by MacLaurin's theorem, we have

$$\log(1+e^x) f(x) = f(0) + xf'(0) + \frac{x^2}{2!} f''(0) + \frac{x^3}{3!} f'''(0) + \frac{x^4}{4!} f^{iv}(0) + \dots$$

$$\therefore \log(1+e^x) = \log 2 + \frac{x}{2} + \frac{1}{4} \frac{x^2}{2!} - \frac{1}{8} \cdot \frac{x^4}{4!} + \dots$$

Q.(3) - Obtain by MacLaurin's Theorem, the first four terms of the expansion $e^{x \cos x}$ in ascending powers of x .

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Solution: — Let $f(x) = e^{x \cos x}$

$$\Rightarrow f(0) = e^0 = 1.$$

Successively differentiating w.r. to x of the function $f(x) = e^{x \cos x}$

$$f'(x) = e^{x \cos x} \cdot (\cos x - x \sin x)$$

$$\Rightarrow f'(0) = 1 \cdot (1 - 0) = 1$$

$$f''(x) = e^{x \cos x} \cdot (\cos x - x \sin x)^2 + e^{x \cos x} (-2 \sin x - x \cos x)$$

$$\Rightarrow f''(0) = 1 (1 - 0)^2 + 1 \cdot (-0 - 0) = 1$$

$$\text{Again } f'''(x) = e^{x \cos x} (\cos x - x \sin x)^3 + 2e^{x \cos x} \cdot \left(\frac{6 \sin x - x \cos x}{x^2}\right)$$

$$x (-2 \sin x - x \cos x) + e^{x \cos x} (\cos x - x \sin x)$$

$$x (-2 \sin x - x \cos x) + e^{x \cos x} (-2 \cos x - \sin x + x \sin x)$$

$$\Rightarrow f'''(0) = 1 - 3 = -2.$$

∴ By MacLaurin's Theorem

$$e^{x \cos x} = 1 + x + \frac{x^2}{2!} - \frac{2}{3!} x^3$$

$$\text{i.e. } e^{x \cos x} = \underline{1 + x + \frac{x^2}{2} - \frac{x^3}{3}} + \dots$$

By

Dr. Brikrama Singh
Associate Professor
Dept. of Maths
Sher Shah College, Sardarni.
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